[Total marks: 60

(03 hours)

N. B.:

1. Attempt any two questions from question numbers 1, 2, 3 and any two questions from question numbers 4, 5, 6.
2. Figures to the right indicate full marks.
3. Simple non-programmable calculator is allowed.

Q.1 (a) Define a standard Brownian motion $\{B(t), t \geq 0\}$. Show that $\{Y(t), t \geq 0\}$ is a martingale where $Y(t) = exp\left\{\theta B(t) - \frac{\theta^2 t}{2}\right\}$. Hence show that $B^2(t) - t$ is a martingale. (10)

(b) State the optional stopping theorem and use it to show that, (05)

$$E\left\{exp\left[\frac{c\left(X(T) - \mu T\right)}{\sigma} - \frac{c^2 T}{2}\right]\right\} = 1$$

where $X(t) = \sigma B(t) + \mu t$ and T is a stopping time.

Q.2 (a) State $MA(q)$ process. Show that it is stationary. Obtain ρ_k, autocorrelation function of order k for $MA(q)$ process. (05)

(b) Determine the range of values for b for which the following process is stationary. (10)

$$X_t = -2bX_{t-1} + b^2 X_{t-2} + e_t$$

where $\{e_t\}$ is white noise process with $E(e_t) = 0$ and $var(e_t) = 1$.

Q.3 (a) Define renewal process $\{N(t); t \geq 0\}$ and prove elementary renewal theorem. (10)

(b) Show that $\{N(t), t \geq 0\}$ is approximately normally distributed. Find it$'$s mean and variance. (05)

Q.4 (a) Explain the following terms (05)

i. type I censoring.
ii. type II censoring.
iii. random censoring.

(b) Derive the Kaplan-Meier estimator of survival function. Derive its approximate variance. (10)

Q.5 (a) All members of a mortality study are observed from birth. Some leave the study by means other than death. At the time of third death t_3 there was one death $d_3 = 1$. At the time of fourth death t_4, there were three $d_4 = 3$ deaths. The Kaplan-Meier estimates of survival function were, $\hat{S}(t_3) = 0.65$, $\hat{S}(t_4) = 0.50$ and $\hat{S}(t_5) = 0.25$. Between t_4 and t_5 six observations were censored. Assuming no observations were censored at times of death, determine d_5, number of deaths at t_5. (06)

(b) Suppose a life time random variable T has the following probability density function, (06)

$$f(t) = \begin{cases} \lambda e^{\phi t} e^{-\dfrac{\lambda}{\phi}\left(e^{\phi t}-1\right)} & ; \; t > 0; \; \lambda > 0; \; \phi > 0 \\ 0 & ; \; \text{otherwise} \end{cases}$$

Derive the coordinates of horizontal and vertical axis in Q-Q plot used to examine whether given random right censored data follow above distribution.

(c) Show that Kaplan-Meier estimator of survival function is discontinuous from left side and hence obtain size of discountinuity. (03)

Q.6 (a) Explain what you mean by proportional hazards model. Obtain survival function and hazard function under this model. (04)

(b) Derive the log-likelihood function for the given data under random censoring. (06)

(c) Obtain log-likelihood function under Cox proportional hazards model when there is only one failure at a time. (05)

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